A comparison of recursive filter and spectral methods for digital correction of pressure measurements distorted by tubing response

T. M. Huang and R. G. J. Flay
The University of Auckland, New Zealand

1. INTRODUCTION

The present investigation is focussed on two methods for correcting pressure signals for distortions introduced by the tubes used to convey the surface pressure to the transducer. This is because it is important to have unity gain and linear phase shift up to at least 200Hz [1]. The digital correction methods investigated were the so-called spectral method which is applied in the frequency domain [2], and a recursive filter method which operates in the time domain. This is to replace the traditional "restrictors" which are inserted into the tube [3] in order to eliminate the resonant effects from the tube.

The Aerodynamics Laboratory in the Mechanical Engineering Department is developing a new electronically scanned pressure system which will record signals from up to 512 pressure transducers at up to 1000 Hz. It was hoped to avoid the use of restrictors in such a large system.

2. THEORY

To regenerate the original pressure signal using a digital correction method requires the physical characteristics of the tube to be formulated into a mathematical function. Then the original pressure signal can be found by applying the inverse of that function. In all that follows, we will take *x(t)* as representing the pressure signal at the surface of the model, and *y* as the distorted signal which is measured by the pressure transducer. In our analysis, we assume that the action of the tube is that of a constant parameter linear system with a weighting function $h(\tau)$ and a frequency response function $H(f)$, (see Fig. 1) where *f* is frequency Hz [4].

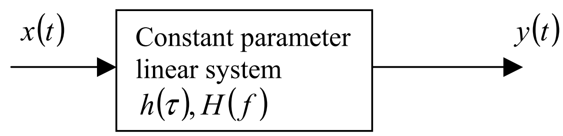

Fig. 1 Block diagram of the theoretical model of the tubing system.

In order to determine *H(f)* it is necessary to measure *x* and *y* simultaneously during a "calibration" of the tube. Then during actual tests, the desired signal *x* can be recovered from the measured signal *y*.

2.1 Spectral Analysis

The spectral approach to correcting the signal distortion due to the tube requires finding the transfer function *H(f),* and applying the correction in the frequency domain. In measurements, one records a time history of the pressure signal from a particular transducer. This signal is then transformed into the frequency domain by taking the Fourier transform of the signal. If one knows the transfer function $H(f)$, then the signal is corrected by applying the following equation.

$$X(f) = Y(f)H(f)^{-1} \qquad (1)$$

The corrected time history is obtained by taking the inverse Fourier transform of the result, namely,

$$x(t) = IFT(X(f)) \qquad (2)$$

where *IFT* is read as the inverse Fourier transform.

2.2 Recursive Filter Method

The general relationship between the input *x(t)* and the output *y(t)* of a linear filter is given by the convolution integral, equation (3).

$$y(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau \qquad (3)$$

where $h(\tau)$ is the weighting function of the filter. The frequency response function of the filter, $H(f)$ is the Fourier transform of $h(\tau)$.

$$H(f) = \int_{-\infty}^{\infty} h(\tau)e^{-j2\pi f\tau}d\tau \qquad (4)$$

The design of a digital filter requires finding weighting functions h_k, where k is time step, to simulate $h(\tau)$. It is not necessary for the filter to be physically realisable. For example, it is not required that $h(\tau)$ be zero for $t<0$, since the data can be stored in the computer and then run backwards to filter the data in reverse order. This feature is used in the present research.

A non-recursive digital filter is one where the output y depends only on the inputs x at various time intervals, e.g.

$$y_n = \sum_{k=1}^{M} h_k\left(x_{n+k} + x_{n-k}\right) \quad n = 1,2,...,N \qquad (5)$$
$$t = k\Delta t, \quad k = 1,2,...,M$$

Δt is the time step between consecutive samples.

A recursive digital filter is one where the output results not only from a finite sum of input terms, but also by using previous outputs as inputs. A simple standard type of recursive filter is given by

$$y_n = cx_n + \sum_{k=1}^{M} h_k y_{n-k} \qquad (6)$$

The advantage of recursive filters over non-recursive filters, is that useful results can be obtained with a very few weights. For example a second order system has a transfer function which can be described by

$$y_n = \gamma x_n + \alpha y_{n-1} + \beta y_{n-2} \qquad (7)$$

where γ, α and β are constants.

3. EXPERIMENTAL SETUP

A schematic diagram of the experimental setup for the research is shown in Fig. 2. A model with two pressure taps 2mm apart facing upstream was placed in the wind tunnel. Large blocks were placed upstream in order to generate a large amount of turbulence, and thus to have a large fluctuating pressure signal, with frequencies up to 300Hz. The reference transducer was connected to a short 40mm long tube to minimise signal distortion. This transducer was assumed to measure the actual pressure signal $x(t)$. Four different lengths of longer tube (0.5m, 0.75m, 1.0m and 1.5m) were used to connect the other pressure tap to the second transducer. This transducer measured the distorted signal, $y(t)$. All tubing had a nominal internal diameter of 1.5mm. Considerable attention was paid to eliminating extraneous "noise" from the signals, due to transducer vibration etc.

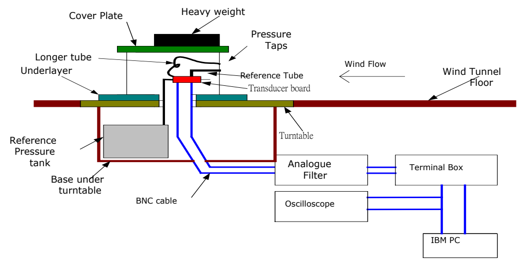

Fig. 2 Schematic layout of experimental set-up

For some of the tests, the fluctuating pressures at the pressure taps were generated by directing a jet of turbulent air at the taps, so as not to tie up the wind tunnel. The pressure transducers were identical, Honeywell XSCL4C, and were calibrated prior to use.

4. DATA COLLECTION AND ANALYSIS

x and y data were recorded simultaneously during test runs. As the objective was to investigate the frequency response of the tubing up to 300 Hz, sampling was carried out at 1000Hz and the signal was low-pass filtered at 400Hz. Each block consisted of 16384 samples collected over a period of 16.4 seconds. Generally 4 or more blocks were used to formulate the transfer functions. The data were collected by a 486 PC via a Metrabyte 12-bit A/D board using standard software.

4.1 Spectral Analysis

Most of the analysis was implemented using MATLAB. The transfer function $H(f)$ was obtained by firstly taking the Fourier transform of each time history (x and y), and then determining the ratio of the cross-spectral density of x and y to the spectral density of the surface pressures (x).

Note that $H(f)$ is complex, and contains both the gain and the phase.

4.2 Recursive Filter Method

As mentioned in Section 2.2, a recursive equation consists of feedback from the calculated signal, and the required number of coefficients depends on the order of the equation. If these coefficients are known then the output can be calculated from the input. In the present case it is the objective to find these coefficients so that the surface pressure could be obtained from the measured pressures. MATLAB allows the user to test recursive equations of various orders. The most useful recursive equation is the one with the smallest number of coefficients which enables the user to recover the desired signal.

4.2.1 Assumed second order system

This section illustrates how we find the coefficients for the pressure tubing, assuming that the frequency response function is second order, equation (7).

Rearranging (7) yields

$$\gamma x_n = y_n - \alpha y_{n-1} - \beta y_{n-2} \tag{8}$$

and introducing new coefficients we get

$$x_n = ay_{n-2} + by_{n-1} + cy_n \tag{9}$$

Given time histories of x and y, the coefficients a, b and c can be determined using a system of matrices, as shown below. Expanding (9) we can write in matrix form,

$$\begin{bmatrix} x_3 \\ x_4 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} y_1 & y_2 & y_3 \\ y_2 & y_3 & y_4 \\ \vdots & \vdots & \vdots \\ y_{n-2} & y_{n-1} & y_n \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} \tag{10}$$

which can be written in the condensed form

$$\{X\} = [A]\{C\}. \tag{11}$$

To convert [**A**] to a square matrix, we pre-multiply by its transpose,

$$[A]^T\{X\} = [A]^T[A]\{C\}. \tag{12}$$

and then to eliminate the pre-multiplying matrices on **C**, we pre-multiply them by their inverse, which results in the following.

$$\{C\} = \left[[A]^T[A]\right]^{-1}[A]^T\{X\} \tag{13}$$

This is often referred to as least squares elimination.

The MATLAB procedures follow those shown above, except that the matrices become larger as the number of coefficients in the recursive equation is increased.

The system identification toolbox in MATLAB is set up in such a way that it can only handle expressions such as (7), where the output y_n relies on the input x_n at the same time, and previous values of the output y_{n-k}. A novel aspect of this study was the reversal of both the input and output time series, and then the investigation to find a, b and c in the following equation

$$x_{n-2} = ay_{n-2} + by_{n-1} + cy_n \tag{14}$$

which requires the future output y to find the present input x. This may seem unrealistic, however, in the tubing system, it is not unreasonable to suppose that previous pressures at the open end of the tube will have some influence on the pressure at the transducer at a later time, after being modified by the tubing system. Results corresponding to this formulation will be referred to as reverse time order henceforth.

4.3 Model Simulation and Validation

The effectiveness of the correction procedures was determined by using x and y time histories that were independent from those used to derive the coefficients in the correction procedures. Two measures were used to test the performance of the models. The standard deviation of the error between the corrected signal and the measured reference signal was one. The other compared the relative errors in estimating the peak pressures.

5. RESULTS AND DISCUSSION

Slightly different results were obtained depending on whether the source of the pressure fluctuations was the wind tunnel or the turbulent air jet. It is believed that these differences are due to differences in background noise and vibration.

5.1 Recursive Filter Method

The effect of increasing the number of terms in the recursive equation was examined. It was found that there was no advantage in going beyond 9 terms in the recursive equation. It was also found that the output y could be computed from x using a digital version of equation (3) with great accuracy. However, when the recursive equation was inverted to enable x to be calculated from y, the equation became unstable, and was unable to be used for correction. When the analysis was reversed, and the longer tube measurement was set as the input and the reference tube measurement was set as the output, the estimation process did not have sufficient accuracy, and a phase shift often occurred. At this point some lateral thinking was introduced, and the data were analysed in reverse order. It was found that by doing this, the phase distortion was eliminated and the accuracy was improved. In wind engineering it is often the peak pressures which are of most importance. The effectiveness of the reverse order correction method can be seen in Fig. 3 for a 1.5m long tube.

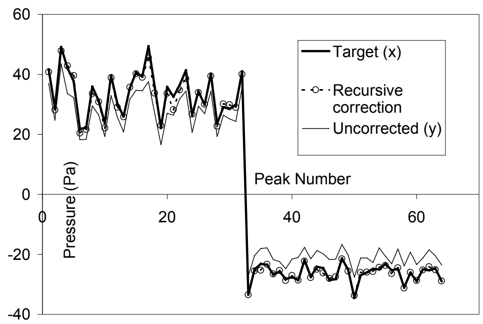

Fig. 3 Effect of recursive correction using reverse time order, on measured peak.

The percent error in the peak estimation is reduced from 18% to 3% by the correction. The improvement reduces as the tube is shortened.

5.1 Spectral Correction Method

The magnitude of the measured transfer function is given in Fig. 4 for the various tube lengths. The results are similar to those in [2].

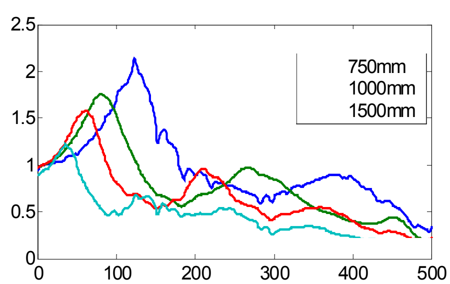

Fig. 4 Magnitude of transfer function for various tube lengths

The spectral method was better than the recursive equation method for reducing the standard deviation of the error for the whole time series, and has similar performance for the peaks. This can be seen in Fig. 5 for a 1.5m long tube.

Overall the results show that both methods give much the same accuracy of the correction. Computationally, the recursive method is superior, since it requires only 9 coefficients, whereas the spectral method requires two FFTs. The recursive equation is more convenient, as it can be applied, along with normal data analysis packages, in the time domain to determine the peak pressure coefficients, and other statistics that are required.

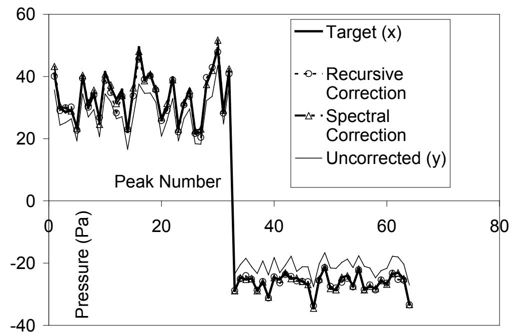

Fig. 5 Effect of different correction methods on measured peak

6. CONCLUSIONS

Two correction methods have been tested and validated

Both correction methods gave significant improvements over the uncorrected signal for both the standard deviations of the time histories, and the errors in the peaks.

The recursive equation was found to be accurate with no more than 9 coefficients.

The effect of the corrections became larger as the tube length increased

7. REFERENCES

[1] Holmes, J.D. “Effect of frequency response of peak pressure measurements”, J. Wind Eng. Ind. Aero. Vol.17, 1984.

[2] Irwin, H.P.A.H., Cooper, K.R. and Girard, R. “Correction of distortion effects caused by tubing systems”, J. Ind. Aero, 5 1979.

[3] Holmes, J.D. and Lewis R.E. “Optimisation of Dynamic pressure meas systems 1”, J. Wind Eng. And Ind. Aero., Vol 25, 1987.

[4] Bendat, J.S. and Piersol, A.G. “Random data: Analysis and measurement procedures”, John Wiley and Sons, 1971.